# Faculty of Science & Environment
# M.G.C.G.V., Chitrakoot, Satan (M.P.)
# M.Sc. (Mathematics)

SEM-I

| SUB CODE | SUBJECT | CREDIT |
|---|---|---|
| SMM-501 | Advanced Abstract Algebra-I | 4+0 |
| SMM-502 | Real Analysis-I | 4+0 |
| SMM-503 | Topology | 4+0 |
| SMM-504 | Complex Analysis-I | 4+0 |

SEM-II

| SUB CODE | SUBJECT | CREDIT |
|---|---|---|
| SMM-505 | Advanced Abstract Algebra-II | 4+0 |
| SMM-506 | Real Analysis-II | 4+0 |
| SMM-507 | Computer Programming<br>(Theory + Practical) | 2+2 |
| SMM-508 | Complex Analysis-II | 4+0 |

SEM-III

| SUB CODE | SUBJECT | CREDIT |
|---|---|---|
| SMM-601 | Functional Analysis | 4+0 |
| SMM-602 | Analytical Mechanics and Calculus of Variation | 4+0 |
| SMM-603 | Optional- (Any One)<br>Numerical Analysis<br>Mathematical Statistics | 4+0 |
| SMM-604 | Optional-(Any one)<br>Integral Equations<br>Advanced Discrete Mathematics-I | 4+0 |

SEM-IV

| SUB CODE | SUBJECT | CREDIT |
|---|---|---|

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|---|---|---|
| SMM-605 | General Measure and Integration Theory | 4+0 |
| SMM-606 | Partial Differential Equation | 4+0 |
| SMM-607 | Optional- (Any one)<br>Probability and Measure<br>Mathematical Modeling<br>Advanced Discrete Mathematics-II | 4+0 |
| SMM-608 | Optional- (Any one)<br>Operation Research,<br>General Relativity & Cosmology<br>Tensor and Differential Geometry | 4+0 |

## Semester-I

### Paper – I: Advanced Abstract Algebra – I

#### Unit – I

Automorphism and Inner automorphism of a group G. The groups Aut(G) and Inn(G). Automorphism group of a cyclic group. Normalizer and Centralizer of a non-empty subset of a group G. Conjugate elements and conjugacy classes. Class equation of a finite group G and its applications.

#### Unit – II

Derived group (or a commutator subgroup) of a group G. Perfect groups. Zassenhau's Lemma. Normal and Composition series of a group G.Scheier's refinement theorem. Jordan Holder theorem. Composition series of a groups of order $p^n$ and of Abelian groups. Cauchy theorem for finite groups. II- groups and p-group Sylow II –subgroups and Sylow p-subgroups. Sylow's I, II and III theorems. Application of Sylow theory to groups of smaller orders.

#### Unit – III

Characteristic of a ring with unity. Prime fields Z/pZ and Q. Field extensions. Degree of an extension. Algebraic and transcendental elements. Simple field extensions. Minimal polynomial of an algebraic element. Conjugate elements. Algebraic extensions. Finitely generated algebraic extensions. Algebraic closure and algebraically closed fields. Splitting fields. Finite fields. Normal extensions.

**Unit – IV**

Separable elements, separable polynomials and separable extensions. Theorem of primitive element. Perfect fields. Galois extensions. Galois group of an extension. Dedekind lemma Fundamental theorem of Galois Theory. Frobeniusautomorphism of a finite field. Klein's 4-group and Diheadral group. Galois groups of polynomials. Fundamental theorem of algebra.

**Unit –V**

Solvable groups Derived series of a group G. Simplicity of the Alternating group $A_n (n \geq 5)$. Non solvability of the semmetric group $S_n$ and the Alternating group$A_n (n \geq 5)$. Roots of unity Cyclotomic polynomials and their irreducibility over Q Radicals extensions. Galois radical extensions. Cyclic extensions. Solvability of polynomials by radicals over Q. Symmetric functions and elementary symmetric functions. Construction with ruler and compass only.

**Recommended Books:**

1. I.D. Macdonald : The theory of Groups
2. P.B. Bhattacharya :Abstract Algebra
3. S.K. Jain & S.R. Nagpal : Basic Abstract Algebra (Cambridge University press 1995 )

**Reference Books-**

1. Vivek Sahai and Vikas Bist: Algebra (Narosa pub. House, New Delhi)
2. I.S. Luther and I.B.S. Passi: Algebra Vol. 1 Groups (Narosa pub. House, New Delhi)
3. I.N.Herstein: Topics in Algebra (Wiley Eastern Pvt. Ltd. New Delhi)

4. Surjit Singh and Quazi Zameeruddin : Modern Algebra (Vikas Publishing House 1990)

5. John B. Fraleigh: A First Course in Abstract Algebra, Pearson India Education, New Delhi (Seventh Edition) 2014.

## Paper -II: Real Analysis –I

### Unit-I

Definition and existence of Riemann Stieltjes integral, properties of the integral, integration and differentiation, the fundamental theorem of integral calculus, integration by parts, integration of vector-valued functions, Rectifiable curves.

### Unit-II

Pointwise and uniform convergence, Cauchy criterion for uniform convergence, Weir strass M-test, Abel' s test and Dirichlet's test for uniform convergence, uniform convergence and continuity, uniform convergence and Riemann Stieltjes integration.

### Unit-III

Uniform convergence and differentiation, existence of a real continuous nowhere differentiable function, equicontinous families of functions, Weir strass approximation theorem.

### Unit-IV

Functions of several variables: linear transformations, Derivative in an open subset of $R^n$. Chain rule, Partial derivatives, directional derivatives, the contraction principle, inverse function theorem, Implicit function theorem, Jacobians, extremum problems with constraints, Lagrange's multiplier method, Derivatives of higher order, mean value theorem for real functions of two variables, interchange of the order of differentiation. Differentiation of integrals.

### Unit-V dddddd

Power Series: Uniqueness theorem for power series. Abel's and Tauber's theorem. Taylor's theorem, Exponential & Logarithm functions.

Trigonometric functions. Fourier series, Gamma function. Integration of differential forms: Partitions of unity, differential forms, stokes theorem.

**Recommended Books:**

'Principles of mathematical Analysis' by Walter Rudin (3rd Edition) McGraw-Hill, 1976.

**Reference Books:**

1. T.M. Apostol, Mathematical Analysis, Narosa Publishing House, New Delhi, 1985.

2. Gabriel Klambauer, Mathematical Analysis, Marcel Dekkar, Inc. New York, 1975.

3. A.J. White, Real Analysis; an introduction. Addison-Wesley Publishing Co., Inc., 1968.

4. E. Hewitt and K. Stromberg Real and Abstract Analysis, Berlin, Springer, 1969,

5. Serge Lang. Analysis I & II. Addison-Wesley Publishing Company Inc. 1969

## Paper –III: Topology

### Unit-I

Definition and examples of topological spaces, Neighborhoods, Neighborhood system of a point and its properties, Interior point and interior of a set, interior as an operator and its properties. definition of a closed set as complement of an open set, limit point (accumulation point) of a set, derived set of a set, definition of closure of a set as union of the set and its derived set, Adherent point (Closure point) of a set, closure of a set as set of adherent (closure) points, properties of closure, closure as an operator and its properties, boundary of a set, Dense sets. A characterization of dense sets. Base for a topology and its characterization, Base for Neighborhood system, Sub base for a topology.

### Unit-II

Relative (induced) Topology and subspace of a topological space. Alternate methods of defining a topology using 'properties' of 'Neighborhood system', 'Interior Operator', 'Closed sets', Kuratowski closure operator and 'base'. First countable, second countable and separable spaces, their relationships and hereditary property. About countability of a collection of disjoint open sets in a separable and a second countable space, Lindel of theorem. Comparison of Topologies on a set, about inter Unit and union of topologies, infimum and supremum of a collection of topologies on a set, the collection of all topologies on a set as a complete lattice (scope as in theorems)

**Unit-III**

Definition, examples and characterizations of continuous functions, composition of continuous functions, Open and closed functions, Homeomorphism, embedding. Tychonoff product topology in terms of standard (defining) subbase, projection maps, their continuity and openness. Characterization of product topology as the smallest topology with projections continuous, continuity of a function from a space into a product of spaces. TO, T1, T2,Regular and T3 separation axioms, their characterization and basic properties i.e. hereditary property of TO, T1, T2, Regular and T3 spaces, and productive property of T1 and T2 spaces.

**Unit IV**

Quotient topology w.r.t. a map. Continuity of function with domain a space having quotient topology, About Hausdorffness of quotient space. Completely regular and Tychonoff (T 3, 1/2), spaces, their hereditary and productive properties. Embedding lemma, embedding theorem. Normal and T4 spaces: Definition and simple examples, Urysohn's Lemma, complete regularity of a regular normal space, T4 implies Tychonoff, Tietze's extension theorem (Statement only). (Scope as in theorems 1-7, Chapter 4 of Kelley's book given at Sr. No.1).

**Unit – V**

Compactness: Definition and examples of compact spaces, definition of a compact subset as a compact subspace, relation of open cover of a subset of a topological space in the sub-space with that in the main space, compactness in terms of finite inter Unit property (f.i.p.), continuity and compact sets, compactness and separation properties, Closeness of compact subset, closeness of continuous map from a compact space into a Hausdorff space and its consequence, Regularity and normality of a compact Hausdorff space.

**Books:**

1. Kelley, J.L.: General Topology.

2. Munkres, J.R.: Topology, Second Edition, Prentice Hall of India/ Pearson.

3. James R. Munkres: Topology, Pearson India Education Inc. New Delhi (2022)

## Paper –IV: Complex Analysis – I

### Unit – I

Algebra of complex, the complex plane, polynomials, transcendental functions such as exponential, trigonometric and hyperbolic functions. Analytic functions, Cauchy-Riemann equations, Power series, its convergence, radius of convergence, examples, sum and product, differentiability of sum function of power series, property of a differentiable function with derivative zero. Exponential-z and its properties, log z power of a complex number (z), their branches with analyticity. Path in a region, smooth path, p.w. Smooth path.

### Unit –II

Contour, simply connected region. Multiply connected region, bounded variation, total variation, complex integration, Cauchy Goursat theorem, Cauchy theorem for simply and multiply connected domains Index or winding number of a closed curve with simple properties Cauchy integral formula. Extension of Cauchy integral formula for multiple connected domain.

### Unit –III

Higher order derivative of Cauchy integral formula. Gauss mean value theorem Morera's theorem. Cauchy's inequality. Zeros of an analytic function, entire function, radius of convergence of an entire function, Liouville's theorem, Fundamental theorem of algebra, Taylor's theorem.

**Unit –IV**

Maximum modulus principle, Minimum modulus principle. Schwarz Lemma. Singularity, their classification, pole of a function and its order. Laurent series, Cassorati- Weiertrass theorem Meromorphic functions, Poles and zeros of Meromorphic functions. The argument principle, Rouche's theorem, inverse function theorem.

**Unit –V**

Residue: Residue at a singularity, residue at a simple pole, residue at infinity. Cauchy residue theorem and it's calculate certain integrals. Definite ($\int_0^{2\pi} f(\cos\theta,\sin\theta)$)$d\theta$,$\int_{-\infty}^{\infty} f(x)dx$, integral of the type $\int_0^{\infty}\sin mx\, dx$ or $\int_0^{\infty} f(x)\cos mx dx$, poles on the real axis, integral of many valued functions.

**Books recommended**

1. Ahlfors, L.V.: Complex Analysis, McGraw-Hill Book Company, 1979
2. Churchill, R.V. and Brown, J.W. : Complex Variables and Applications Mcgraw-Hill Pub., Comp.,1990
3. Conway, J.B.: Functions of One Complex Variables, Narosa Pub. New Delhi 2000

## Semester -II

## Paper –V: Advanced Abstract Algebra –II

**Unit-I**

Similar linear transformation. Invariant subspaces of vector spaces. Reduction of a linear transformation to triangular form. Nilpotent transformations Index of nilpotency of a nilpotent transformation. Cyclic subspace with respect to a nilpotent transformation. Uniqueness of the invariants of a nilpotent transformation.

**Unit –II**

Primary decomposition theorem Jordan blocks and Jordan canonical forms cyclic module relative to a linear transformation. Companion matrix of a polynomial $R_s$) Rational Canonicals form of a linear transformation and its element ary divisior. Uniqueness of the elementary division.

**Unit –III**

Modules, submodules and quotient modules. Module generated by a non-empty subset of an R module. Finitely generated modules and cyclic modules. Idempotents. Homomorphism of R modules. Fundamental theorem of homomorphism of R-modules.

**Unit-IV**

Direct sum of modules. Endomorphism rings End. (M) and Ends (M) of a left R-module M. Simple modules and completely reducible modules (semi-simple modules). Finitely generated free modules. Rank of a finitely generated free module. Submodules of free modules of finite rank over a PID Endomorphism ring of a finite direct sum of modules. Finitely generated modules.

**Unit-V**

Ascending and descending chains of sub modules of an R-module. Ascending and Descending change conditions (A.C.C. and D.C.C.) Noetherian modules and Noetherian rings, finitely co-generated modules. Artinian modules and Artinian rings Nil and nilpotent ideals Hilbert Basis Theorem. Structure theorem of finite Boolean rings Wedeerburn-Artin theorem and its consequence

**Recommended Books:**

**1**. Basic Abstract Algebra P.B. Bhattacharya S.R. Jain and S.R. Nagal

**2**. Theory of Groups: L.D. Macdonald 3. Topics in Algebra: I.N. Herstein

**4.** Group Theory: W.R. Scott

**5.** Algebra. Vol.I, II: Ramji Lal, Shail Pub. Allahabad

## Paper –VI: Real Analysis- II

**Unit –I**

Lebesgue outer measure, elementary properties of outer measure.

Measurable sets and their properties. Lebesgue measure of sets of real numbers, algebra of measurable sets, Borel sets and their measurability, characterization of measurable sets in terms of open, closed, F and G sets existence of a non-measurable set.

**Unit –II**

Approximation of measurable functions by sequences of simple functions. Measurable functions as nearly continuous functions. Bored measurability of a function.

**Unit –III**

Almost uniform convergence. Egoroff's theorem, Lusin's theorem, convergence in m F. Riesz theorem that every sequence which is convergent in measure has an almost everywhere convergent subsequence. The Lebesgue Integral: Shortcomings of Riemann integral, Lebesgue integral of a bounded function over a set of finite measure and its properties, Lebsegue integral as a generalization of the Riemann integral.

**Unit-IV**

Bounded convergence theorem, Lebesgue theorem regarding points of discontinuities of Riemann integrable functions. Integral of a non-negative function, Fatou's lemma, Monotone convergence theorem, integration of series, the general Lebesgue integral. Lebesgue convergence theorem.

**Unit – V**

Differentiation and Integration Differentiation of monotone functions. Vitali's covering lemma, the four Diniderivatives. Lebesgue differentiation theorem, functions of bounded variation and their representation as difference of monotone functions.

**Recommended Books**:

1. Real Analysis: HL Royden, PHI, New Delhi, 1999

**Reference Books:**

**1**. G.de Barra, Measure theory and integration, Willey Eastern lid, 1981 **2**. PR.Halmos, Measure Theory, Van Nostrans, Princeton, 1950
**3**.1.P.Natanson. Theory of functions of a real variable, Vol. 1. Frederick

Ungar Publishing Co. 1961

4. R.G. Bartle. The elements of integration, John Wiley & Sons, Inc. New York, 1966

5. KR. Parthsarthy, Introduction to Probability and Measure, Macmillan Com, Of India Delhi 1977

6. P.K. Jain and V.P. Gupta, Lebesgue measure and integration, New age lat. P. Lad ND, 1986

## Paper- VII: Computer Programming (Theory)

**Unit –I**

Numerical constants and variables, arithmetic expressions; input/output, conditional flow, looping: Logical expressions and control flow: functions, subroutines, arrays.

**Unit –II**

Format specifications: strings: array arguments, derived data types $C^{++}$ Data types, and keywords decision making, looping, functions, arrange and pointers, dynamic memory allocation.

**Unit –III**

Practicals for $C^{++}$

1 Sum the fibonauli series for n terms and write a programme to implement it in $C^{++}$.

2 WAP to find Wheather any given number is armonstrong number or not.

3. Write a programme to sum the individual digits of a number.

4. Write a programme to find prime factors of given number.

**Unit-IV**

Basic functions of MATLAB (Theory)

**Unit-V**

MATLAB PRACTICALS

1. Solutions of simultaneous linear equations.
2. Solution of algebraic transcendental equations.
3. Inversion of matrices.
4. Numerical differentiation and integration.
5. Solution of ordinary differential equations.
6. Statistical problems on central tendency and dispersion.
7. Fitting of curves by least square method.

**Recommended Books:**

1. E. Balaguruswamy: Object oriented Programming in $C^{++}$. TMH Pub. N.D.

## Paper-VIII: Complex Analysis-II

### Unit-I

Bilinear or Mobious Transformation, Mapping or transformation, Jacobian of an transformation, Linear transformation, Product of resultant of two Mobious transformation, Simple Geometric transformation, Mobious transformation as the resultant of an Even Number of Inversions, Fixed point of Mobious transformation, Cross ratio, Preservance of cross-ratio under Mobious Transformations, Special Bilinear Transformation.

### Unit-II

Introduction, Conformal Mapping, Sufficient and Necessary condition for W =f(z) to represent a conformal mapping, Type of the Transformation in Conformal Mapping.

### Unit-III

Spaces of analytical function and their completeness, Hurwitz's theorem, Montels theorem, Riemann mapping theorem, Infinite products, Weierestrass theorem, Factorization of sine function.

### Unit-IV

Gamma function and its properties, functional equations for gamma function, integral version of gamma function. Riemann-zeta function, Riemann's functional equtional, Runge's theorem, Mittage -Lefflers throrem.

### Unit-V

Analytic continuation, uniqueness of direct analytic continuation, uniqueness of analytic continuation along a curve, Power series method of analytic continuation, Schwarz reflection principal.

### Text Book:

1. Ahlfors L.V., Complex analysis: McGraw-Hill Book Comp.1979

2. Churchill. R.V. & Brown, J.W., Complex Variables & Applications, Mc Graw -Hill.1990

3. Conway J.B., Functions of one Complex Variables: Narosa Pub.2000 N.D

### Reference Book-

1. Priestly, H.A., Introduction to Complex Analysis, Claredon Press, Oxford,1990
2. Liang-shin Hann & Bernard Epstein, Classical Complex Analysis, Jones and Bartlett Publishers International, London, 1996.

## Semester-III

## Paper-IX: Functional Analysis

### Unit-I

Normed linear spaces, Banach spaces and examples, subspace of a Banach space, completion of a normed space, quotient space of a normed linear space and its completeness, product of normed spaces, finite dimensional normed spaces and subspaces, equivalent norms, compactness and finite dimension, F. Riesz slemma. Bounded and continuous linear operators, differentiation operator, integral operator. Bounded linear extension, linear functionals, bounded linear functionals, continuity and boundedness, definite integral, canonical mapping, linear operators and functionals on finite dimensional spaces, normed spaces of operators, dual spaces with examples.

**Unit –II**

Hahn-Banach theorem for real linear spaces, complex liner spaces and normed liner spaces application to bounded linear functionals onC[a,b], Riesz-representation theorem for bounded linear functionals onC[a,b], adjoint operator, norm of the adjoint operator. Reflexive spaces, uniform boundedness theorem and some of its applications to the space of polynomials and Fourier series.

**Unit –III**

Strong and weak convergence, weak convergence in $l_p$ , convergence of sequences of operators, uniform operator convergence, strong operator convergence, weak operator convergence, strong and weak convergence of a sequence of functions. Open mapping theorem, bounded inverse theorem, closed linear operators, closed graph theorem, differential operator, relation between closedness and boundedness of a linear operator.

**Unit –IV**

Inner product spaces,Hilbert spaces and their examples, Pythagorean theorem, Apolloniu's .Identity. Schwarz inequality, continuity of innerproduct, completion of an inner product space, subspace of a Hilbert space, orthogonal complements and direct sums, projection theorem. Characterization of sets in Hilbert spaces whose space is dense.

**Unit –V**

Orthonormal sets and sequences,Bessel's inequality, series related to orthonormal sequences and sets. Total(complete) orthonormal sets and sequences, Parseval's identity, separable Hilbert spaces. Representation of functionals on Hilbert spaces. Rieserepresentation theorem for bounded linear functionals on a Hilben space, sesquilinear form, Riesz representation theorem for hounded sesquilinear forms on a Hilbert space. Hilbert adjoint operator, its existence and uniqueness, properties of Hilbert adjoint operators, self adjoint, unitary, normal, positive and projection operators.

**Recommended Books**:

E.Kreyszig: Introductory Functional Analysis with Applications, John Wiley and Sons, New York,1978

**Reference Books**

1. G.F. Simmons: Introduction to Topology and Modern Analysis, McGraw-Hill New York, 1963

2. C. Goffman and G. Pedrick: First Course in Functional Analysis, PHI, N.D.1987

3. G Bachman and L. Narici: Functional Analysis.Academic Press 1966 4. L.A. Lustenik and V.J. Sobolev: Elements of Functional Analysis, Hindustan Pub. Co.N.D1971

5. J.B. Conway: A Course in Functional Analysis. Springer-Verlag 1990

## Paper –X: Analytical Mechanics and Calculus of variations

### Unit –I

Variation of a functional, Euler-Lagrange equation, necessary and sufficient conditions for extrema. Variational methods for boundary value problems in ordinary and partial differential equations.

### Unit –II

Variational derivative, invariance of Euler's equations, natural boundary conditions and conditions. Conditional extremum under geometric constraints and under integral constraints Varishle end points Free and constrained systems, constraints and their clanification Generalized coordinates Holom and Non-Holonomic systems Sclerotic and Rheonomic gms Generalind Potential, Possible and virtual displacements, ideal constraints.

### Unit-III

Lagrange's equations of first kind Principle of viral displacements D'Alembert's principle Holonomic Systems independent coordinates, generalized for Lagrange's spations of wind sind Uniqueness of solution Theorem on variation of total Energy Potential, Gyeps and dissipative forces. Lagrange's equations for potential forces equation for conservative fields.

**Unit-IV**

Hamilton variables. Donkin's theorem, Hamilton canonical equations. Routh's equations, cyclic coordinates. Poisson's identity, Jacobi-poissons theorem. Hamilton principle, Second form of hamiltone principle. Poincare Carton integral invariant, Whittaker's equations, Jacobi equations, Principle of least action.

**Unit –V**

Canonical transformations, free canonical tormations theorem Method of separation of variables for solving Hamilton-Jacob qua ating the Canical character of a transformation Lagrange brackets Condition of canestical character of tratatio terms of Lagrange brackets and Poison brackets Simplicial nature of t Jabian transformations Invariance of Lagrange brackets and Poisson brackets under conical transformations.

**Recommended Books:**

1. F. Gantmacher: Lectures in Analytic Mechanics, Khosla Pub.House New Delhi

2. H. Goldstein: Classical Mechanics (2edition), Narosa Pub. H, New Del

3.I.M. Gelfand and S.V. Fomin: Calculus of Variations, Prentice Hall N.D.

4. Francis B. Hilderbrand: Methods of Applied Mathematics, Prentice Hall N.D.

5. Narayan Chandra Rana Pramod Sharad Chandra Joag: Classical Mechanics, Tata McGraw Hill, 1991

6. Louis N, Hand and Janet D. Finch: Analytical Mechanics, Cambridge University Press, 1998

**Optional –I**

## Paper-XI: Numerical Analysis

### Unit –I

Numerical solutions of algebraic equations, method of iteration and Newton-Raphson method, rate of convergence, Solution of system of liner algebraic equations using Gauss elimination and Gauss- Seidel methods,

### Unit-II

Finite differences, operators, Newton's forward and backward difference interpolation formula, Newtons divided difference, Lagrange, Hermite and spline interpolation.

### Unit-III

Numerical differentions and numerical quadrature, Trapezoidal, Simpson's rule, Weddle's rule .

### Unit-IV

Numerical solutions of Ordinary differential equations using Picard, Taylor's , Euler,modified Euler and Runge-Kutta methods.

### Unit-V

Numerical solutions of partial differential equations, Boundary value Problems, Laplace, Heat, Wave Equcation.

### Recommended Books:

1. S.S. Shasti.
2. G. Shanker Rao; Numerical Analysis,New Age International (P) Ltd., Pub. Second Edition 2002

## Optional-I

## Paper –XII: Mathematics Statistics

### Unit-I

Random distribution: preliminaries, Probability density function, Probability models, Mathematical Expectation, Chebyshev's Inequality. Conditional probability, Marginal and conditional distributions, Correlation coefficient, Stochastic independence.

**Unit-II**

Frequency distributions: Binomial, Poisson, Gamma, Chi-square, Normal, Bivariate normal distributions.

**Unit-III**

Distributions of functions sampling. Transformations of variables, discrete and continuous; t & F distributions, Change of variable technique, Distributions of order, Moment-generating function technique, other distributions and expectations.

**Unit-IV**

Limiting distributions: Stochastic convergence, Moment generating function, Related theorems intervals: Random intervals, Confidence intervals for mean, differences of means and variance, Bayesian estimation.

**Unit-V**

Estimation & sufficiency: Point estimation, sufficient statistics, Rao-Blackwell Theorem, Completeness. Uniqueness, Exponential PDF. Functions of parameters, Stochastic independence.

**Recommended Books:**

1. R.V. Hogg & AT. Craig: Introduction to Mathematical Statistics, Amerind Pub Co. Pvt. Ltd., New Delhi, 1972. (Chapters 1 to 7)

2. S.C. Gupta, V. K. Kapoor: Fundamentals of Mathematical Statistics, Sultan Chand & Sons,N.D. 2007.

## Optional-II

## Paper- XIII: - Integral Equations

**Unit- I: General Concepts of Integral Equation**

Introduction, Physical Problem, Abel’s Problem, Initial and Boundary value Problems, Definitions, Non linear integral Equations, Singular Integral Equations, Integro-Differential Equation, Types of Solutions, Formula For Differentiation of an Integral Involving Parameter, Formula, Transformation of Differential into integral Equation and Vice Versa.

**Unit-II: Solution of Volterra’s Integral Equations**

Solution of Volterra’s Integral Equations by the method of successive substitution and approximation, Resolvent karnel of Volterra’s Integral Equations, Determinations of resolvent kernals when k(x,t) is a polynomials, Volterra’s Integral Equation of first kind, Solution of non-linear Volterra’s Integral Equation.

**Unit-III: Solution of Fredholm Integral Equation**s

Solution of Fredholm Integral Equations by the method of successive substitutions and approximations, Determination of the conditions of convergences, Reciprocal Functions, Orthogonal kernels, Method of the Fredholm Determinants, Linear Integral Equation with Degenerate kernels, Non-linear Fredholm Integral Equations with Degenerate kernels.

**Unit-IV: Integral Transform Method Integral Equations**

Definition Laplace transform, Table of Laplace transform, Theorems of Laplace transform, Inverse Laplace transform, Table of Inverse Laplace transform, Convolution Theorem, Properties of Convolution, Theorems of Inverse Laplace transform, Fourier Transform, Theorems of Fourier Transform, Solution of Volterra Integral Equation by the aid of Laplace transform,

**Unit-V: Application of Integral Transform in Boundary value Problems**

Equations of Different type, Definition, Results of Integral Transform,

**Reference Books:**


1. A.B. Chandramouli: Integral Equations with Boundary Value Problems, Shiksha Sahitya Prakashan, Meerut.(2009)
2. Brijendra Singh etal.: Integral Equations, Golden Valley Pub. Agra(2012)


**Optional-II**

**Paper-XIV:-Advanced Discrete Mathematics-I**

**Unit-I**

Graphs, Konisberg seven bridges problem. Finite and infinite graphs Incidence vertex. Degree of a vertex Isolated and pendant vertices Null graphs homesphism of graphs Subgraphs, walks, path and circuits Connected and disconnected graphs Components of a graph. Euler graphs. Hamiltonian paths and circuits.

**Unit-II**

The traveling salesman problem. Trees and their properties. Pendant vertices in a tree Rooted and binary tree. Spamming tree and fundamental circuits Spanning tree in a weighted graph (Chapter1.2.3 of the book given at Sr. No. 1)

**Unit-III**

Cut-Sets and their properties Fundamental circuits and cur-sets Connectivity and separability. Network flows Planner grapho Kuratowski's two graphs. Representation of planner graphs. Euler formula for planner graphs. Vector space associated with a graph. Basis sectors of a graph Circuit and cut-set subspaces. Interunit and joint of $W_c$ and $W_s$.

**Unit-IV**

Incidence matrix A (G) of a graph G, Submatrices of a group G, Circuit matrix. Fundamental circuit matrix, and its rank. Curser matrix, path matrix and adjacency matrix of a graph. (Chapter 4 Theorems 5.1 to 5.6 of chapter 5, chapter 6 & 7 of the book given at Sr. No 1).

**Unit-V**

Introduction, the difference calculus The difference operator falling factorial power binomial coefficient summation definition properties and examples, Abel's summation formula Generating functions, Euler's motion formula, Bernoulli polynomials and examples approximate summation.

**Recommended Books**

1. W.G. Kelley and A.C. Peterson: Difference Equations; an Introduction with Applications. Academic Press, Harcourt 1991

2. Calvil Ahlbrandt and Allan C., Peterson. Discrete Hamiltonian systems,

Defference Equations, Continued Fractions & Ricati Equation, Kluwer Botson 1996.

## Semester -IV

## Paper-XV: Probability and Measure

### Unit-I

Binomial Random Variables-Poisson Theorem Interchangeable Events Bernoulli, Borel theorem Central limit theorem for binomial random variables. Large deviations.

### Unit-II

Sums independent random variables Three series those of large numbers stopping times, elementary renewal theorem, optional stopping.

### Unit-III

Conditional expectation, conditional independence, introduction martingales.

### Unit-IV

Distribution & characteristic functions, Central limit theorems.

### Unit-V

Limit theorems for independent and wishes-Laws of numbers, law of the tested the Dominated Ergesic theorems, Masins of random walks.

### Recommend Books:

1. Y.S. Chow & H. Teicher, Probability theory- Independence Interchangeability, Martingales, springer/Narosa pub. 1979

2. K. L. Chung: Elementary Probability theory with Stochastic Processes, Springer int. Student Edition/Narosa Pub. 1975

3. Richard A. Johnson & Dean W. Wichern: Pearson Education Inc 2013

## Paper –XVI: General Measure and integration theory

### Unit-I

Measures, some properties of measures, outer measures, extension of measures, uniqueness of extension, completion of a measure, the LUB of an increasingly directed family of measures (Scope as in the Units 3-6, 9-10 of Chapter I of the book 'Measure and Integration by S.K. Berberian) Measurable functions, combinations of measurable functions, limits of measurable functions. Localization of measurability, simple functions (Scope as in Chapter 2 of the book "Measure and Integration by S.K. Berberian).

**Unit-II**

Measure spaces, almost everywhere convergence, fundamental almost everywhere, convergence in measure, fundamental in measure, almost uniform convergence, Egoroff's theorem, Riesz-Weyl theorem (Scope as in Chapter 3 of the book 'Measure and Integration by S.K. Berberian) Integration with respect to a measure: Integrable simple functions, non-negative integrable functions, integrable functions, indefinite integrals, the monotone convergence theorem, mean convergence (Scope as in Chapter 4 of the book 'Measure and Integration by S.K. Berberian)

**Unit –III**

Product Measures: Rectangles, Cartesian product of two measurable spaces, measurable rectangle. Units, the product of two finite measure spaces, the product of any two measure spaces, product of two - finite measure spaces; iterated integrals. Fuhini's theorem, a partial converse to the Fubin's theorem.

**Unit-IV**

Signed Measures: Absolute continuity, finite singed measure, contractions of a finite signed measure, purely positive and purely negative sets, comparison of finite measures, Lebesgue decomposition theorem. a preliminary Radon-Nikodym theorem, Hahn decomposition, Jordan decomposition.

**Unit-V**

Upper variation, lower variation, total variation, domination of finite signed measures, the Radyon-Nikodym theorem for a finite measure space, the Radon-Nikodym theorem IV for a finite measure space.

**Recommended Books:**

S.K.Berberian: Measure and Integration. Chelsea Publishing Company. New York, 1965.

1. H.L. Royden Real Analysis, Prentice Hall of India. 3 Edition, 1988

2. G.de Barra: Measure Theory and Integration, Wiley Eastern Ltd., 1981.

3. P.R.Halmos: Measure Theory, Van Nostrand, Princeton, 1950.

4. L.K.Rana: An Introduction to Measure and Integration, Narosa Publishing House, Delhi, 1997. 5. R.G. Bartle: The Elements of integration. John Wiley & Sons, Inc. New York 1966.

## Optional-I

## Paper-XVII: Partial Differential Equation

### Unit-I

Existence and uniqueness of solutions of initial value Problems for first order ordinary differential equations, Singular Solutions of first order ODEs, System of first order ODEs.

### Unit-II

General Theory of homogenous and non-homogenous linear ODEs, Variation of Parameters, Sturm-Liouville's boundary value problem, Green's Function.

### Unit-III

Origin of PDEs, Lagrange and Charpit Methods for solving first order PDEs, Cauchy Problem for first order PDEs.

### Unit-IV

Homogenous and non-homogenous linear PDEs with Constant coefficients, Classification of second order PDEs, General Solution of higher order PDEs with Constant Coefficients.

### Unit-V

Method of separation of variables for Laplace, Heat and Wave Equations.

**Reference Book:**

1. M.D. Raisinghania: Ordinary and Partial Differential Equations, S.Chand and Company PVT. Ltd. New Delhi(2016)

## Optional-I

## Paper –XVIII: Mathematical Modeling

### Unit-I

The process of Applied Mathematics; mathematical modeling: need, techniques, classification and illustrative; mathematical modeling through ordinary differential equation of first order; qualitative solutions through sketching.

### Unit-II

Mathematical modeling in population dynamics, epidemic spreading and compartment models, mathematical modeling through systems of ordinary differential equations, mathematical modeling in economics, medicine, arm -race, battle.

### Unit-III

Mathematical modeling through ordinary differential equations of second order. Higher order (linear) models. Mathematical modeling through difference equations Need, basic theory. Mathematical modeling in probability theory, economics, finance, population dynamics and genetics.

### Unit-IV

Mathematical modeling through partial differential equations, simple models, mass-balance equations, variational principles, probability generating function, traffic flow problems, initial & boundary conditions. Newtons equations of motion. Methematical modeling. Continuum approach.

### Unit-V

External flow: Fluid Dynamics Forces Acting on Moving Bodies. Flying and Swimming Blood flow in heart, lungs, arteries and veins. Micro and Microcirculation.

Repiratory Gas Flow the laws of thermodynamics, Molecular, Diffusion. Mechanisms in membrances and multiphasic structure. Mass Transport in capillaries.

**Book Recommended**:

1. JN. Kapoor: Mathematical Modeling. Wiley Eastern Limited, 1990 (Relevant portions, mainly from Chapters 1 to 6.)

2. Y. C. Fung: Biomechanics, Springer-Verlag. New York Inc. 1990

## Optional-I

## Paper-XIX: Advanced Discrete Mathematics-II

### Unit-I

Partially ordered sets and lattices. Lattice as an algebraic system. Sublattices .Isomorphism of lattices. Distributive and modular lattices. Lattices as intervals .Similar and projective intervals .Chains in lattices. Zassenhaus's Lemma and Schreier Theorem.

### Unit-II

Composition chain and Jordan Holder Theorem. Chain conditions. Fundamental dimensionality relation for modular lattices. Decomposition theory for lattices with ascending chain conditions, i.e. reducible and irreducible elements. Independent elements in lattices.(Relevant portion of the chapter 7 and chapter 12 of the books given at Sr. No. 2 & 3).

### Unit-III

Points (atoms) of a lattice. Complemented lattices. Chain conditions and complemented lattices. Boolean algebras. Conversion of a Boolean algebra into a Boolean ring with unity and vice versa. Direct product of Boolean algebras. Uniqueness of finite Boolean algebras. Boolean functions and Boolean expressions. Application of Boolean algebra to switching circuit theory (Relevant portion of the chapter 7 and chapter 12 of the books given at Sr. No. 2 & 3).

### Unit-IV

Stability Theory: Initial value Problems for Linear systems, eigen values, eigen vectors and spectral radius, Caylay-Hamilton Theorem, Putzer algorithm. Solution of nonhomogeneous system with initial conditions, Stability of linear systems, stable subspace theorem and example. Stability of non-linear system, chaotic behaviour.

### Unit-V

The Z-Transform, definition. Properties, initial and final value Theorem. Convolation Theorem, Solving the initial value problems, Volterrasummation equation and Fredholmsummation equation by use of Z-Transform. Asymptotic Methods: Introduction, Asymptotic Analysis of Sums, and examples. Asymptotic behavior of solutions of homogeneous linear equations, Poincare's Theorem, Perron Theorem (Statement only), non-linear equations.

### Recommended Books

1. W.G. Kelley and A.C. Peterson: Difference Equations; an Introduction with Applications. Academic Press, Harcourt 1991

2. Calvil Ahlbrandt and Allan C., Peterson. Discrete Hamiltonian systems, Defference Equations, Continued Fractions & Ricati Equation, Kluwer Botson 1996.

## Optional-II

## Paper-XX: - Operational Research

### Unit-I

Dynamic Programming - Nature of Dynamic Programming (DP). Bellman's principle of optimality in DP, DP algorithm, mathematical formulation of multistage model, the recursive operation approach, Application of DP in Linear Programming. Integer Programming: types of integer programming problem, cutting plane method (Gomory technique), construction of Gomory's constraints, Graphical interpretation of cutting plane method, cutting plane algorithm, Fractional cut method the branch and bound method.

### Unit-II

Dynamic Programming - Nature of Dynamic Programming (DP). Bellman's principle of optimality in DP, DP algorithm, mathematical formulation of multistage model, the recursive operation approach, Application of DP in Linear Programming. Integer Programming: types of integer programming problem, cutting plane method (Gomory technique), construction of Gomory's constraints, Graphical interpretation of cutting plane method, cutting plane algorithm, Fractional cut methodthe branch and bound method.

**Unit-III**

Algebraic method for the solution of general game, equivalence of the rectangular matrix games and linear programming, fundamental theory of game theory, limitation of game theory, solution of rectangular game by singular method, matrix method for (nxn) game.

**Unit-IV**

Nonlinear Programming-Definition and examples of non-linear programming. Mobi- Tucker theory: Kuhn-Tucker (K-T) optimality conditions, K-T first order necessary optimality conditions, K-T. Second order optimality conditions. Lagrange's method. Economic interpretation of multipliers-Wolf duality theorem on non-linear programming. Quadratic programming. K-T conditions for Quadratic programming problems, Wolf modified simplex method. Beale's method, separable, convex and non-convex programming.

**Unit-V**

Inventory model classification of inventory models, Determinsite inventory model (DIM), Basic Economic-order quantity (EOO) models, EOQ model with uniform rate of demand infinite production rate and having no shortage EOQ model with uniform rate of demand in different production cycles, infinite production rate & having non shortage, EOQ with finite replenishment DIM with shortage. Fixed Time Model, EOQ with finite production, EOQ with price break, E00 with one price break, single multi-item deterministic inventory model. Queuing models: classification of queuing models, solution of queue models, model I (M/M/1): (/FCFS) model il (General Erlongqueuing model, model III M/M/1): (N/FCFS), Network (PERT/CPM), schedule chart (Gantt Bar Chart), difference between CPM and PERT, Network components, construction of the Network diagram, CPM analysis**.**

### *Recommend Text:*

1. G.Hadley: Linear Programming

2. C.W. Churchman et al.: Introduction to Operations Research

3. B.S. Goel& S.K. Mittal: Operations research

4. D. Gross & C.M. Harris: Fundamentals of Queuing Theory

5. V.K. Kapoor Operations Research, Sultan Chand & Sons. 6. Kanti Swarup: Operations Research, Sultan Chand & Sons

## Optional-II

## Paper-XXI:-General Relativity and Cosmology

### Unit-I

Review of the special theorem of relativity and the Newton's theory of gravitation, Principle of equivalence and general covariance, geodesic principle.

### Unit-II

Newton’s approximation of relativistic equations of motion, Einstein's field equation and its Newton’s approximation. Schwarzschild external solution and its isotropic form.

### Unit-III

Planetary orbits and analogues of kaplers law in general relativity. Advance of perihelion of planet. Bending of light rays in a gravitational field. Gravitational redshift of spectral lines.

### Unit-IV

Mach s principle. Einstein modified field equation with cosmological terms. Static cosmological models Einstein and De-sitter, their derivation properties and comparison with the actual universe.

## Unit-V

Hubble's law, corological principles, weiyll's postulate, derivation of Robertson walker metric.Hubble and deceleration parameters, Redshift redshift, versus distance relation. Angula size versus redshifi relation and source counts in Robertson-walker-space-time.

### Recommended Books:

1. H. Stephan, Genral Relativity: An Introduction of the theory of the gravitational field Cambridge University Press, 1982.

2. A.S.Eddington, the mathematical theory of Relativity,Cambridge Univ.P.1965.

3. J.V Narlikar General Relativity and Cosmology: The Macmilan Company of india Lt.1978.

4. B.F.Shutz: A first in General Relativity, Cambridge University Press, 1990.

5. S.R. Roy & Raj Bali: Theory of Relativity, Jaipur Pub. H. Jaipur 1987.

## Optional-II (Any one)

## Paper-XXII: - Tensor and Differential Geometry

## Unit-I: Tensor Algebra

Covariant and contravariant vectors, Tensors of second order, Mixed Tensor of type (p,q), Zero Tensor, Tensor Field, Algebra of Tensor, Equality of Two Tensors, Symmetric and Skew-Symmetric Tensors, Outer multiplication and Contraction, Inner multiplication, Quotient Law of Tensors, Reciprocal Tensor of a Tensor, Relative Tensor, Cross Product or Vector product of two vectors.

## Unit-II: Tensor Calculus

Introduction, Riemannian Space, Christoffel Symbols and their proparties, covariant differentiation of Tensors, Riemann-Christoffel curvature Tensor.

## Unit-III

General curve theory, curves, arc length and Linear Motion, curvature, integral curves, Planar Curves, the fundamental equations, the rotation index, three interesting result, convex curves, space curves, the fundamental equations characterizations of space curves, closed space curves.

**Unit-IV**

Basic surface theory, surfaces, tangent spaces and maps, the first fundamental form, curvature of surfaces, curves on surface, the Gauss and Weingarten maps and equations, the Gauss and Mean curvatures, principal curvatures, ruled surfaces, surface theory, generalized and abstract surfaces, curvature on abstract surfaces, the Gauss and Codazzi equations, the Gauss-Bonnet theorem, topology of surfaces, closed and convex surfaces.

**Unit-V**

Geodesics and matric geometry, Geodesics, mixed partials, shortest curves, short geodesics, distance and completeness, isometries, constant curvature, comparison results.

**Recommend Books:**


1. Peter Petersen: Classical Differential Geometry(text book)


**Reference Books:**


1. U.C. De etal.: Tensor Calculus, Narosa Pub. House Pvt. Ltd. New Delhi(2015)
2. T.J. Willmore: Differential Geometry, Oxford, Clarendon press, 1959.