Master of Science (Mathematics)

Course Outline and Marks Distribution

M.Sc. (Mathematics)

Centre for Distance learning & Continuing Education

Mahatma Gandhi Chitrakoot Gramodaya Vishwavidyalaya, Chitrakoot Distt. Satna (M.P.)

Mahatma Gandhi Chitrakoot Gramodaya Vishwavidyalaya, Chitrakoot Distt. Satna (M.P.)

Syllabus for M.Sc. (Mathematics) First Year

S.N.	Name of Paper	Code	Int. Marks	Ext. Marks	Total Marks	Credit
1.	Algebra	SMM-501	20	80	100	4+0
2.	Topology	SMM-502	20	80	100	4+0
3.	Mathematical Methods	SMM-503	20	80	100	4+0
4.	Advanced Statistics	SMM-504	20	80	100	4+0
5.						

Mahatma Gandhi Chitrakoot Gramodaya Vishwavidyalaya, Chitrakoot Distt. Satna (M.P.)

M.Sc. (Mathematics) Final Year

S.N.	Name of Paper	Code	Int. Marks	Ext. Marks	Total Marks	Credit
1.	Real and Complex Analysis	SMM-601	20	80	100	4+0
2.	Differential Geometry	SMM-602	20	80	100	4+0
3.	Advance Mechanics	SMM-603	20	80	100	4+0
4.	Fluid Mechanics	SMM-604	20	80	100	4+0
5.	Functional Analysis	SMM-605	20	80	100	4+0

Contents of the Papers
M.Sc. (Mathematics) First Year
First Paper- Algebra

Unit -1

Normal series, Schreier's theorem, Composition series, Existence and uniqueness of composition series, Direct decomposition, Remak–Krull–Schmidt theorem, Group action, Class equation of an action, Sylow theorems, Application of Sylow theorems to groups of certain orders using counting arguments, Commutator calculus, Commutator subgroup of permutation and matrix group.

Unit-2

Solvable groups, Nilpotent groups, Finite nilpotent groups, Frattini subgroup, Wielandt theorem, Finite b-group, Burnside basis theorem, Special and extra special p-groups, Classification of groups of order p^3, Free groups, Presentation of groups, Coxeter enumeration method, Nelson Schreerer theorem (Statement only).

Unit-3

Factorization in rings, Unique Factorization Domains, Gauss theorem, Einstein's irreducibility criterion, Principal ideal domains, Euclidean Domains, Chinese remainder theorem.

Unit-4

Modules—elementary notion, Netherlands Modules and rings, Hilbert basis theorem, Free, projective and injective modules, Exact sequence, Five lemma, Short exact sequence, Split exact sequence, Modules over Principal ideal domain, Structure theorem of finitely generated modules over PID's, Classification of finite abelian group.

Unit-5

Hom and Tensor, Adjointness of Hom and Tensor, Left exactness of Hom and right exactness of Tensor, Elementary computation of Hom and Tensor products, Tensor algebra, Exterior product, Exterior, Symmetric and Divided power algebras.

M.Sc. (Mathematics) First Year
Second Paper- Topology

Unit -1 Elements of set Theory, Metric Spaces

Sets, Functions and relations, Real Numbers, Products and Co-products, Finite and Infinite sets, Countable and Uncountable, Well-Ordered sets, Partially ordered sets, The maximum principle and Zorn's lemma. The real line $\mathbb{R}$, Metric, Euclidean spaces, Some important inequalities, Bounded and Unbounded spaces, Some important metric spaces, Spheres (or Balls), Open sets, Closed sets, Neighbourhood, Accumulation points, Adherent points, Interior, exterior and boundary of a set, Dense and Non-dense sets, Sequences and sub-sequences in metric spaces, Cauchy's sequences, Complete metric spaces, Completeness and contracting mappings, Some complete metric spaces, Completion of a metric space.

Unit-2 Topological spaces, Normal spaces and manifolds

Topological spaces, Order topologies, The Product topology, The subspace topology, Closed sets and limit points, Continuous function, The quotient topology, Metric Topology, Connected spaces, Compact spaces, Locally compact spaces and Alexandroff, Compactification, Countability Axioms, Separation Axioms, Normal spaces, Second countable regular spaces and the Urysohn metrization theorem, Completely regular spaces and the stones–čech compactification, Manifolds

Unit-3 Algebraic Topology Basics

What is algebraic topology? The fundamental group, Dependence on the base point, Homotopy invariance calculations:
$\pi_1(\mathbb{R})=0$ and $\pi_1(S^1)=\mathbb{Z}$, The Brouwer fixed point theorem, The fundamental theorem of algebra, Categories, Functors, Natural transformations, Homotopy categories and homotopy equivalence, The fundamental groupoid, Limits and Co-limits, The Van kampen theorem, Examples via van kampen theorem The definition of covering spaces, The unique path lifting property, Covering of groupoids, Group actions and orbit spaces, The classification of covering of groupoid, Construction of covering of groupoid, The classification of covering of spaces, The construction of towers of spaces

Unit-4 Graph, Weak Spaces, Cofibrations & Fibratipons

The definition of graphs, Edge paths and trees, The homotopy types of graphs, Covers of graphs and Euler characteristics, Applications to groups, The definition of compactly generated spaces, The category of compactly generated spaces, Cofibrations and Mapping Cylinders, Fibrations and path lifting functions

Unit-5 Advanced Homotopy Theory

cofiber and fiber sequences, Homotopy groups and change of basepoint, Long exact sequences, weak equivalences, CW Complexes and cellular approximation, CW complexes, HELP and the Whitehead theorem, The homotopy excision and Freudenthal theory.

M.Sc. (Mathematics) First Year
Third Paper- Mathematical Methods

Unit -1

Series Solutions of Differential Equations, Bessel's Function, Legendre's Function.

Unit-2

Hyper Geometric Functions, Hypergeometrical Differential Equation and its Solution, Symmetric Property of Hypergeometric Function, Integral Formula for the Hypergeometric Function, Contiguous Hypergeometric Functions, Differentiation of Hypergeometric Functions, The Confluent Hypergeometric Function

Unit-3

Formation of Partial Differential Equation (PDE), Partial Differential Equations of First Order, Lagrange's Linear Equation, Charpit's Method

Unit-4

Second Order Linear Partial Differential Equations with Constant Coefficients, Non-linear Equations of the Second Order, Canonical Forms for two Independent Variables, Separation of Variables, Laplace, Diffusion and Wave Equations in Cartesian, Spherical, Polar and Cylindrical Coordinates, Boundary value problems on transverse vibrations of strings and heat diffusion.

Unit-5

Fourier Transforms & Inverse Transform, Convolution Theorem, Solution of Differential Equations (Using Fourier transforms).

M.Sc. (Mathematics) First Year
Fourth Paper- Advance Statistics

Unit -1

Random variable,Discrete random variable,Continuous random variable, Distribution Function, Mathematical Expectation and theorem on it, Cauchy Schwartz's inequality, Cheby cheff's Inequality, Law of large number, Bernoullie's Law of Large number, Moment generating function, Cumulents, Characteristic function and theorem on it, Central limit theorem.

Unit-2

Continuous probability distribution, Rectangular distribution, Exponential distribution, Gamma distribution, Beta distribution, Cauchy distribution.

Unit-3

Estimation Theory, Point Estimation, Properties of good estimation, Consistency, Unbiasedness, Efficiency, Method of estimation, Cramer Rao Inequality, 95% Confidence for intervals for special case, Confidence intervals – Large Samples.

Unit-4

Testing of Hypothesis, Hypothesis, Null Hypothesis, Alternative Hypothesis, Critical region, Error in Sampling, Level of Significance, Power function of a test, Power of the test, Neyman J. and Pearson, E.S. Lemma, Non parametric methods, Wald–Wolfowitz run test, Median test, Sign test

Unit-5

Finite difference, Factorial Notation, Other difference operator, Interpolation formula for equal interval, Newton's Gregory forward interpolation (with equal interval), Newton's Gregory backward interpolation (with equal interval), Central difference formula, Lagrange's interpolation formula (with unequal space x), Newton's Divided difference interpolation formula, Numerical integration, Trapezoidal rule, Simpson's one third rule, Simpson's one third rule, Simpson's three eight rule

M.Sc. (Mathematics) Second Year
First Paper- Real & Complex Analysis

Unit -1

Definition and existence of Riemann–Stieltjes integral, properties of the integral, Integration and differentiation, the fundamental theorem of calculus, integration of vector valued functions, rectifiable curves.

Unit-2

Rearrangements of the terms of the series, Riemann's theorem.

Sequences and series of functions, point wise and uniform convergence. Cauchy criterion for uniform convergence, Weierstrass M-test, Able's and Dirichlet's test for uniform convergence, uniform convergence and continuity. Uniform convergence and Riemann–Stieltjes integration, Uniform convergence and differentiation, Weierstrass approximation theorem, power series, uniqueness theorem for power series, Able's and Tauber's theorem.

Unit-3

Function and several variable, linear transformation, derivatives in an open subset of R^n, Chain rule, Partial derivatives. Interchange of the order of the differentiation, Derivatives of higher orders. Taylor's theorem, inverse function theorem, Jacobian's extremum problems with constraints, Lagrange's method. De-differentiation of integrals, partitions of unity, Differential forms, Stoke's theorem.

Unit-4

Complex integration. Cauchy–Goursat theorem. Cauchy's integral formula. Higher order derivatives. Morera's theorem. Cauchy's inequality and Liouville's theorem. The fundamental theorem of algebra. Taylor's theorem, maximum modulus principle, Swartz lemma. Laurent's series. Isolated singularities. Meromorphic function. The argument principle. Roche's inverse function theorem.

Unit-5

Residues, Cauchy's residue theorem. Evaluation of integrals, Branches of many valued function with special reference to arg, Z log Z and Z^n. Bilinear transformations, their properties and classifications. Definitions and examples of conformal mappings.

M.Sc. (Mathematics) Second Year
Second Paper- Differential Geometry

Unit -1

Curve in space $\mathbb{R}^3$, Arc length, Tangent, Osculating plane, normal place, principal & Binormals, Curvature & Torsion, Serret–Frenet formula. Frenet's approximation. Osculating circle, Osculating sphere, Spherical indicatrices, involutes & evolutes, Helies Intrinsic equations of a curve.

Unit-2

Surfaces in $\mathbb{R}^3$. Surfaces, surfaces of revolution, helicoids, families of curve on a surface. The first & second fundamental forms of a surface, Principal directions, Lines of curvature, principal curvature and Gaussian curvature. The Dupin indicatrix, normal curvature, conjugate curves, asymptotic lines. Developables, minimal surfaces, rule surfaces.

Unit-3

Christoffel symbols, Gauss & Weingarten formula, Gauss equations, Codazzi Manindi equations, The Riemannian curvature tensor, Geodesics, geodesic on surfaces of revolution, geodesic curvature of a curve.

Unit-4

Differentiable manifolds: differentiable manifolds. Tangent vectors. The differential of a map, the tangent & cotangent bundles. Sub-manifolds, vector fields. Distributions tensors & Differential forms: tensor product, tensor fields and differential forms.

Unit-5

Curvature and torsion tensor. The Koszul connexion, Riemannian metrics, Identities satisfied by the curvature tensor of a Riemannian manifold. Sectional curvature.

M.Sc. (Mathematics) Second Year
Third Paper- Advance Mechanics

Unit -1 Rigid Body

System of particles, Rigid Body, Euler's Theorem, Kinetic energy of rigid body, Equation of motion of rigid body, Euler's dynamical equations

Unit-2 Holonomic and Non-Holonomic System

Generalized Coordinates, Non-Holonomic System Applications, Holonomic Systems with Symmetry, Lagrangian Reduction for non-holonomic system

Unit-3 Generalized Momentum

Generalized Momentum Hamiltonian, Hamilton's equation of motion, Hamilton's equation of motion in different coordinate system, Applications of Hamilton's equation of motion, Small Oscillations, Examples of small oscillations

Unit-4 Hamilton's System

Hamilton's Variational Principle, Principle of least action, Contact Transformations, Hamilton Jacobi Method, Particle Falling Freely, Kepler's problem solution by H–J method, Poisson brackets, Jacobi's Identity

Unit-5 Fluid Mechanics

Law of conservation of momentum, Law of conservation of mass, Energy equation for an incompressible fluid, Kelvin's circulation theorem, Gravity waves, Stream lines, Stream functions, Kelvin's minimum energy theorem, Mean potential over spherical, Theorem of Blasius, Motion of sphere

M.Sc. (Mathematics) Second Year
Fourth Paper- Fluid Mechanics

Unit -1

Motion of Cylinders

Unit-2

Irrotational Motion in Three Dimensions – Sources, Sinks & Doublets; Stoke's Stream Function, Conformal Transformation, Aerofoil and Some Important Theorems

Unit-3

Equation of Motion, Kinematics

Unit-4

Vortex Motion (Rectilinear Vortices), Motion in Two Dimensions

Unit-5

Irrotational Motion in Three Dimensions – Motion of a Sphere, Waves